

IE 383 Lecture 5

Continuous Case - Single Facility

Minimum: minimum total travel distance

User	x	y	(w _i) Demand
A	200	200	75
B	100	500	105
C	250	600	135
D	500	300	60

a). straight-line distance:

$$\min \sum_i w_i d_i^2 \quad d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}$$

$$\min \sum_i w_i (x_i - x)^2 + \sum_i w_i (y_i - y)^2$$

$$\min \sum_i w_i (x_i - x)^2$$

$$\min \sum_i w_i (y_i - y)^2$$

Center of Gravity:

$$x^* = \frac{\sum w_i x_i}{\sum w_i} \quad y^* = \frac{\sum w_i y_i}{\sum w_i}$$

$$x^* = \frac{200 \cdot 75 + 100 \cdot 105 + 250 \cdot 135 + 500 \cdot 60}{75 + 105 + 135 + 60} = 238$$

$$y^* = 444 \quad \text{ANS: } (238, 444)$$

- final solution location tend to be closer to point w/ higher traffic

- d_i^2 magnifies large distances but brings point closer; lops on large distance but less traffic ignores point.

b). Minimum: Rectilinear Distance:

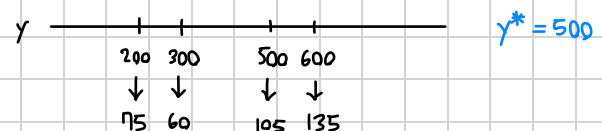
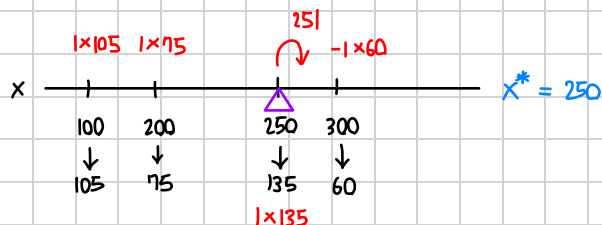
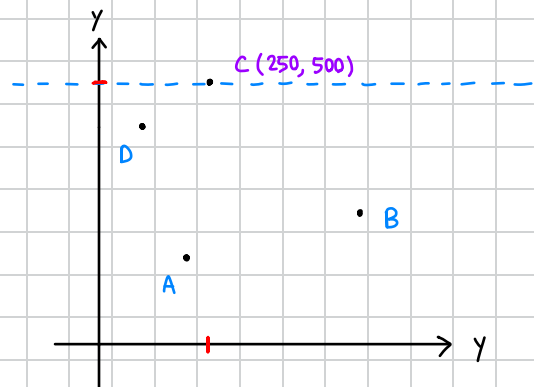
$$\min \sum_i w_i (|x_i - x| + |y_i - y|)$$

$$\left\{ \begin{array}{l} \min \sum_i w_i |x_i - x| \\ \min \sum_i w_i |y_i - y| \end{array} \right.$$

median condition:

$$\sum W = \frac{75 + 105 + 135 + 60}{2} = 187.50$$

$$\left. \begin{array}{l} w_1 \leq \frac{1}{2} \sum W \\ w_2 \leq \frac{1}{2} \sum W \end{array} \right\} \text{possible}$$



- most cases ANS on a coordinate

	1	2	5	7
X		100	50	75
		12	16	20
Y		100	75	50

$X=2$ to $x=5$
all satisfy median condition

only $y=16$ satisfies

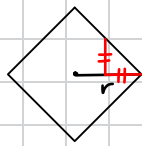
ANS: any point on the line connected by $(2, 16)$ & $(5, 16)$, including the 2 points.
• could be an area w/ 4 points.

Minimax: ignore demand / traffic.

a). Rectilinear Distance:

diamond shape:
(perfect square rotated 45°)

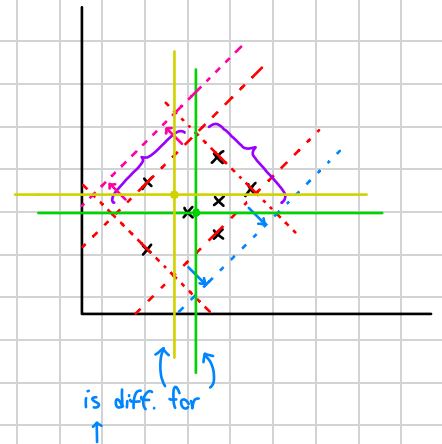
machine	x_i	y_i	$x_i + y_i$	$-x_i + y_i$
1	0	0	0	0
2	4	6	10	2
3	8	2	10	-6
4	10	4	14	-6
5	4	8	12	4
6	2	4	6	2
7	6	4	10	-2
8	8	8	16	0



every point has a distance of r to the center point.

minimize r !

have to make sure all client locations are on the edge or within the shape!



$$C_5 = \max(16, 10) = 16. \quad P_1 = (3, 5) \quad P_2 = (6, 2); \quad \text{max travel dist.} = \frac{C_5}{2} = 8.$$

office to serve those 1
Minimize the maximum
travel distance per
 $C_1 = \min_i (x_i + y_i)$
 $C_2 = \max_i (x_i + y_i)$
 $C_3 = \min_i (-x_i + y_i)$
 $C_4 = \max_i (-x_i + y_i)$
 $C_5 = \max(C_2 - C_1, C_4 - C_3)$
 $P_1^* = \left(\frac{C_1 - C_3}{2}, \frac{C_1 + C_5}{2} \right)$
 $P_2^* = \left(\frac{C_2 - C_4}{2}, \frac{C_2 + C_4 - C_5}{2} \right)$
Max dist(r) = $C_5/2$

Max dist(r) = $C_5/2$
y point on the line connected
by P_1^* and P_2^* is optimal

Minimax: Straight Line

↳ similar approach but circle

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Continuous Case: Single Facility

	Minisum	Minimax	
Rectilinear	Median Condition		} very close to (Location Problems) optimal
Straight Line	Center of Gravity "Minisum" $\sum W_i d_i^2$		

System Analysis (Capacity)

A).

```

    graph LR
      In(( )) -- 100/day --> 1[1]
      1 -- 20 --> 2[2]
      2 -- 5 --> 3[3]
      3 --> Out(( ))
      style 1 fill:#fff,stroke:#000
      style 2 fill:#fff,stroke:#000
      style 3 fill:#fff,stroke:#000
  
```

100/day → 20 → 80/day → 5 → 75/day

max cap. = 75/day (under perfect condition)

utilization: $\frac{\text{machining time}}{\text{work time}}$

→ can have 100% U.R. but w/ surplus 20.

Max input: 75/day

UR #1 : $75/100 = 75\%$

UR #2 : $75/80 = 93.75\%$

UR #3 : $75/75 = 100\% \leftarrow \text{bottleneck}$

If demand = 50/day

UR1 = 50%

UR2 = 3/8

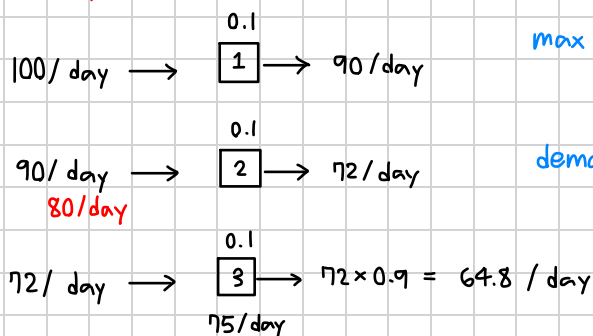
UR3 = 2/3

B). Reverse order

U.R. > 80% = almost perfect operation (non-ideal IRL)

Queue: caused by variation

67.5 / day



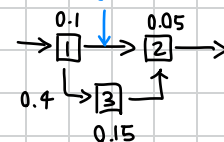
$$\text{max input rate} = \frac{64.8}{0.9 \times 0.9 \times 0.9} = 88.89 / \text{day}$$

$$\text{demand} = 50 / \text{day}, \text{ input} = \frac{50}{0.9 \cdot 0.9 \cdot 0.9}$$

line yield.



Parallel: 0.5

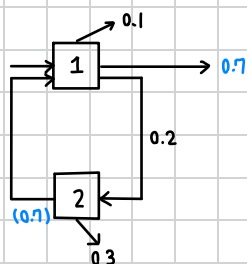


Line yield:

$$1-2 \quad 1-3-2$$

$$0.5 \times 0.95 + 0.4 \times 0.85 \times 0.95$$

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$$\frac{1}{0.7} + \frac{1-2-1}{0.2 \times 0.7 \times 0.7} + \frac{1-2-1-2-1}{0.2 \times 0.7 \times 0.2 \times 0.7 \times 0.7} + \frac{1-2-1-2-1-2-1}{0.2 \times 0.7 \times 0.2 \times 0.7 \times 0.2 \times 0.7 \times 0.7} + \dots$$

$$= 0.7 + (1 + (0.2 \times 0.7) + (0.2 \times 0.7)^2 + (0.2 \times 0.7)^3 + \dots) = 0.7 \left(\frac{1}{1 - 0.2 \times 0.7} \right)$$

Common Ratio

$$r = 0.2 \times 0.7 \quad 0 < r < 1$$

$$= 0.14$$

$$0 < r < 1 \quad S = 1 + r + r^2 + r^3 + \dots \quad (1)$$

$$r \cdot S = r + r^2 + r^3 + r^4 + \dots \quad (2)$$

$$(1) - (2) \quad (1-r)S = 1$$

$$S = \frac{1}{1-r}$$

n^{th} term of a geometric sequence $a, ar, ar^2, \dots$ is,

$$a_n = ar^{n-1}$$

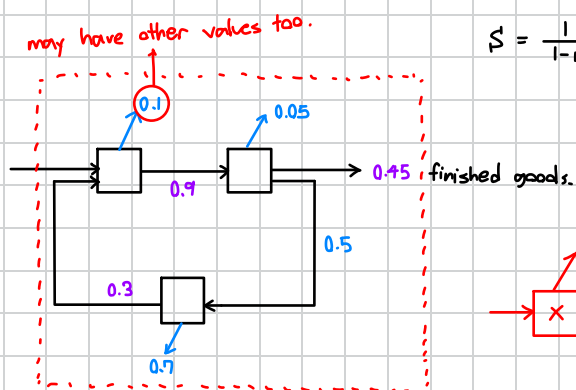
Sum of n term of a finite geometric sequence is,

$$a + ar + ar^2 + \dots + ar^{n-1}$$

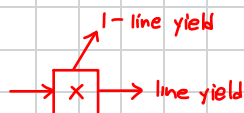
$$= \frac{a(1-r^n)}{1-r} \quad (\text{OR}) \quad \frac{a(r^n-1)}{r-1}, \text{ when } r \neq 1$$

Sum of infinite geometric sequence is,

$$a + ar + ar^2 + \dots = \frac{a}{1-r}, \text{ when } r < 1$$



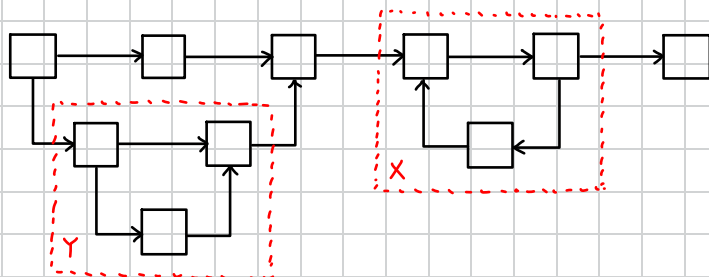
Line Yield:



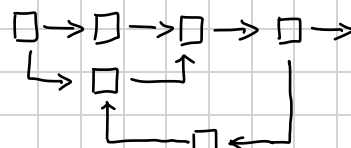
$$\frac{1-2}{0.9 \times 0.45} + \frac{1-2-3-1-2-}{0.9 \times 0.5 \times 0.3 \times 0.9 \times 0.5} + (0.9 \times 0.5 \times 0.3)^2 \times 0.9 \times 0.45 + \dots$$

$$= 0.9 \times 0.45 (1 + 0.9 \times 0.5 \times 0.3 + (0.9 \times 0.5 \times 0.3)^2 + \dots)$$

$$= 0.9 \times 0.45 \left(\frac{1}{1 - 0.9 \times 0.5 \times 0.3} \right) = LY_x$$



cannot simplify / Monte Carlo Simulation

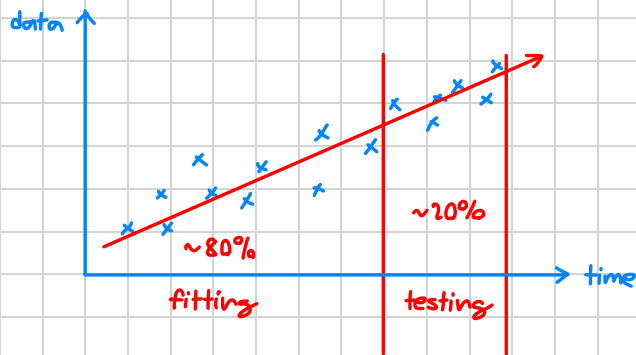


IE383 Lecture 7 Part 2 - Midterm 2 onwards

Forecasting { Qualitative
Quantitative

exponential smoothing

Testing
→ Error analysis



$$e = Y(t) - \hat{Y}(t)$$

= real data - predictive data.
/value

MAD:

$$\left(\sum_{t=1}^N |e(t)| \right) / n$$

Mean absolute deviation

MSE:

$$\left(\sum_{t=1}^N e(t)^2 \right) / n$$

MSE, Mean Square of Error.
n: testing data point

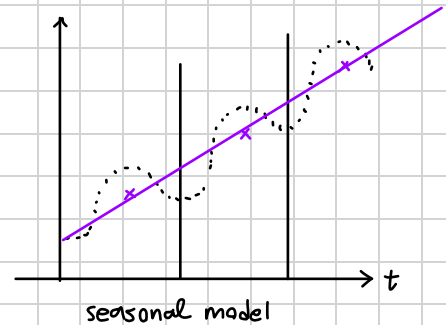
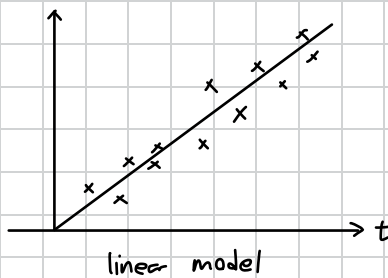
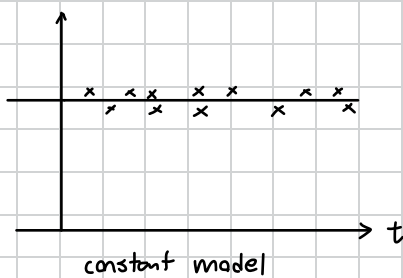
MAP:

$$\left(\sum_{t=1}^N \left| \frac{e(t)}{Y(t)} \right| \right) / n$$

Mean Abs. Percentage.

Fitting Approach:

model - data pattern



Minimize MSE => Linear Regres.

Constant Model $\hat{Y}(t) = a$

$$\hat{a} = \frac{\sum_{t=1}^N Y(t)}{N}$$

Linear Model $\hat{Y}(t) = a \cdot t + b$

slope

$$\hat{a} = \frac{N \cdot \sum t \cdot Y(t) - \sum t \sum Y(t)}{N \cdot \sum t^2 - (\sum t)^2}$$

=> N: # of fitting data

given on
exam 2

intercept

$$\hat{b} = \frac{\sum t^2 \sum Y(t) - \sum t \sum t Y(t)}{N \cdot \sum t^2 - (\sum t)^2}$$

MSE

testing stage
(e.g. 80%)

fully analyzed

t	1	2	3	4	5
Y(t)	10	12	15	18	20

$$\sum t \cdot Y(t) = 1 \times 10 + 2 \times 12 + 3 \times 15 + 4 \times 18 + 5 \times 20$$

$$\sum t = \sum t^2 = 1^2 + 2^2 + \dots + 5^2$$

$$\sum Y(t)$$

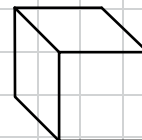
$$\hat{a} = 2.6 \quad \hat{b} = 7.2$$

$$\therefore \text{model: } \hat{Y}(t) = 2.6 \cdot t + 7.2$$

$$t=10, \quad Y(10) = 2.6(10) + 7.2$$

Moving Average

t	$Y(t)$	$N=3$	$N=5$	$N=3$
1	2	}		
2	4			
3	6			
4	8	$\frac{2+4+6}{3}$		$\frac{0.2 \times 2 + 0.3 \times 4 + 0.5 \times 6}{3}$
5	10	$\frac{4+6+8}{3}$		
6	12		$\frac{2+4+6+8+10}{5}$	



N large OR $small = 1$

Δ from real data to predic. is minimized

very little/no memory good for dynamics

N { large : stable
small = 1 : dynamic

$\frac{1+...+100}{100}$
only 1/100 changes

Exponential Smoothing

$$\hat{Y}(t) = \alpha \cdot Y(t) + (1-\alpha) \hat{Y}(t-1)$$

α : weight between two
First-order Smoothed Avg

$0 < \alpha < 1$: Smoothing constant

$Y(t)$: Most recent (real) data

$Y(t-1)$: Momentum

Constant Model - Exponential Smoothing

$$Y'(t) = a(t) + \frac{e(t)}{\text{error}}$$

$$\alpha = 0.2$$

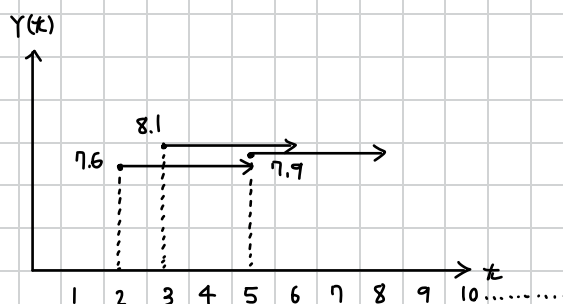
t	$Y(t)$	$\alpha \cdot Y(t)$	$(1-\alpha) \hat{Y}(t)$	$\hat{Y}(t)$
1	8			8
2	6	$0.2 \times 6 = 1.2$	$0.8 \cdot 8 = 6.4$	7.6
3	10	$0.2 \times 10 = 2$	$0.8 \cdot 7.6 = 6.08$	8.1
4	8	$0.2 \times 8 = 1.6$	$0.8 \cdot 8.1 = 6.48$	8.1
5	7	$0.2 \times 7 = 1.4$	$0.8 \cdot 8.1 = 6.48$	7.9

$$\begin{aligned} \hat{Y}(1) &= Y(1) \\ &= \frac{Y(1) + Y(2) + Y(3)}{3} = 8 \end{aligned}$$

$$\hat{Y}(5) = 7.9$$

→ used up 5 data & 7.9 is the best guess

$t = 6, 7, \dots, \infty$ predicted value is 7.9



Linear Programming for Aggregate Planning

Optimal Solution

Cost Parameters and Given Information:

C_H = Cost of hiring one worker

C_F = Cost of firing one worker

C_I = Cost of holding one unit of stock for one period

C_R = Cost of producing one unit on regular time

C_O = Incremental cost of producing one unit on overtime

C_U = Idle cost per unit of production

C_S = Cost to subcontract one unit of production

n_t = Number of production days in period t

K = Number of aggregate units produced by one worker in one day

I_0 = Initial inventory on hand at the start of the planning horizon

W_0 = Initial workforce at the start of the planning horizon

D_t = Forecast of demand in period t

Cost Parameters and Given Information:

W_t = Workforce level in period t

P_t = Production level in period t *everything produced in-house*

I_t = Inventory level in period t

H_t = Number of workers hired in period t

F_t = Number of workers fired in period t

O_t = Overtime production in units *no. of units produced in overtime*

U_t = Worker idle time in units ("undertime") *no. of units that could've been produced in down time*

S_t = Number of units subcontracted from outside

all variables have soln. (at end)

main focus: W_t

Minimizing Total Cost

min $\sum_{t=1}^T (C_H \cdot H_t + C_F \cdot F_t + C_I I_t + C_R \cdot P_t + C_O \cdot O_t + C_U \cdot U_t + C_S \cdot S_t)$

planning horizon (e.g. no. of months)

work force *inventory* *purchases from outside*

s.t.

Work force level:

$$W_t = W_{t-1} + H_t - F_t$$

$$t = 1, \dots, T$$

Inventory level:

$$I_t = I_{t-1} + \underbrace{P_t}_{\text{in-house productions}} + \underbrace{S_t}_{\text{subcontract}} - \underbrace{D_t}_{\text{demand}} \quad t = 1, \dots, T$$

Production / Workforce:

$$P_t = n_t \cdot K \cdot W_t + O_t - U_t$$

$$t = 1, \dots, T$$

all variables are non-negative

$$W_0, I_0,$$

Which variable is not listed in Obj function? A) H_t
B) F_t
C) P_t
✓ D) none of the above

$W_t = ?$

Example:

Hiring Cost: \$500/person

Firing Cost: \$1000/person

Inventory Cost: \$80/unit-month

Production Cost in Regular Hour: \$20/hr

" Overtime: \$50/hr

1 working day: 8hrs/day

Subcontract: \$100/unit

Idle time: \$10/hr

Current work force level: 200 people

" Inventory: 500 units

of working days

t	1	2	3	4	5	6
n_t	20	20	20	20	20	20
D_t	5000	8000	7000	12000	10000	6000

K : 16/day-person

Regular: \$20/hr $\rightarrow$?/unit

$$\frac{\$20 \times 8}{16} = \$10/\text{unit}$$

$C_R = \$10$

$C_0 = \$15 = 25 - 10$

$C_n = \$5$

Overtime: \$50/hr $\rightarrow$?/unit

$$\frac{\$50 \times 8}{16} = \$25/\text{unit}$$

Under time: \$10/hr $\rightarrow$?/unit
(idle)

$$\frac{\$10 \times 8}{16} = \$5/\text{unit}$$

$$\min \sum_{t=1}^6 (500 H_t + 1000 F_t + 80 I_t + 10 P_t + 150 \tau + 5 u_t + 100 S_t)$$

(not both)
 either one
 (not both)
 either one

s.t.

$$\Rightarrow W_t = W_{t-1} + H_t - F_t, \quad t = 1, \dots, 6$$

$$\Rightarrow I_t = I_{t-1} + P_t + S_t - D_t, \quad t = 1, \dots, 6$$

$$I_1 = 500 + P_1 + S_1 - 5000$$

$$I_4 = I_3 + P_4 + S_4 - 12000$$

$$I_2 = I_1 + P_2 + S_2 - 8000$$

$$I_5 = I_4 + P_5 + S_5 - 10000$$

$$I_3 = I_2 + P_3 + S_3 - 7000$$

$$I_6 = I_5 + P_6 + S_6 - 6000$$

$$\Rightarrow P_t = n_t \cdot k \cdot w_t + O_t - u_t$$

$$P_1 = 20 \cdot 16 \cdot W_1 + O_1 - u_1, \quad t = 1, \dots, 6$$

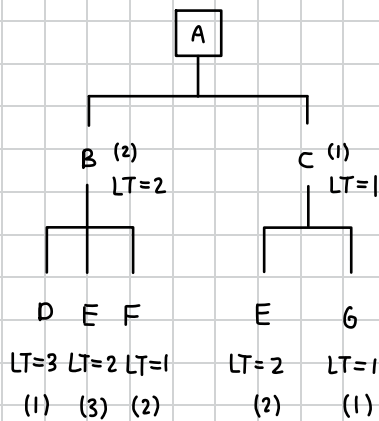
$$\Rightarrow W_t, H_t, F_t, I_t, P_t, O_t, u_t, S_t \geq 0 \quad t = 1, \dots, 6$$

Is it possible to have both H_2, F_2 being positive (non-zero)?
A - yes
B No No could have both zero

Lecture 10: Material Requirement Planning (MRP)

BOM: Bill of materials

Product Tree:



Lead Time (LT)

mfg process
purchase

E's for one A?

$$3 \cdot 2 + 2 \cdot 1 = 8$$

GR: Gross Requirements

SR: Scheduled Receipts (external contractors)

BI: Beginning Inventory

NR: Net Requirement

POR: Planned Order Release

	A											
	LT = 1 day						Lot for Lot					
(day) t	1	2	3	4	5	6	7	8	9	10	11	12
GR	20	15	30	25	12	18	25	20	10	15	10	10
SR	14	10	0	0	0	0	0	0	0	0	0	0
BI	40	34	29	0	0	0	0	0	0	0	0	0
NR	0	0	1	25	12	18	25	20	10	15	10	10
POR	0	1	25	12	18	25	20	10	15	10	10	—

$$GR(B) = 2 \times POR(A)$$

	B											
	LT = 2 days						Lot for Lot					
(day) t	1	2	3	4	5	6	7	8	9	10	11	12
GR	0	2	50	24	36	50	40	20	30	20	20	—
SR	0	0	0	0	0	0	0	0	0	0	0	0
BI	20	20	18	0	0	0	0	0	0	0	0	0
NR	0	0	32	24	36	50	40	20	30	20	20	—
POR	32	24	36	50	40	20	30	20	20	—	—	—

infeasible MRP Table

$$GR(C) = 1 \times POR(A)$$

	C											
	LT = 1 day						Lot for Lot					
(day) t	1	2	3	4	5	6	7	8	9	10	11	12
GR	0	1	25	12	18	25	20	10	15	10	10	—
SR	0	0	0	0	0	0	0	0	0	0	0	0
BI	10											
NR	0	0	16	12	18	25	20	10	15	10	10	—
POR	0	16	12	18	25	20	10	15	10	10	—	—

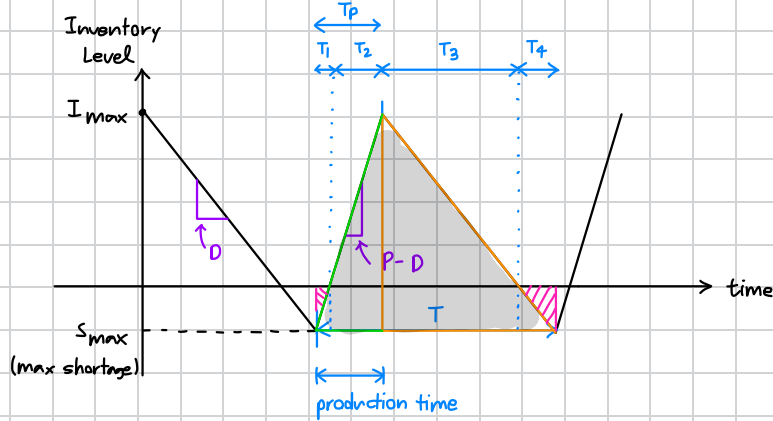
infeasible MRP Table

$$GR(E) = 3 \times POR(B) + 2 \times POR(C)$$

	D											
	LT = 2 days						Lot for Lot					
(day) t	1	2	3	4	5	6	7	8	9	10	11	12
GR	96	104	132	186	170	100	110	90	80	—	—	—
SR	0	0	0	0	0	0	0	0	0	0	0	0
BI	200	104	0	0	0	0	0	0	0	0	0	0
NR	0	0	132	186	170	100	110	90	80	—	—	—
POR	0	132	186	170	100	110	90	80	—	—	—	—

infeasible MRP Table

IE383: Inventory Model



D = demand (linear: assume constant D . rate)

P = production rate (assume constant)

$$P > D$$

→ batch production (MRP: EOQ) → Q : Economic Order Quantity
lot size / batch lot
size of triangle

Variables: Q, S_{max}

Constants: D, P

$$D = \frac{I_{max} + |S_{max}|}{T_3 + T_4}$$

$$P = \frac{I_{max} + |S_{max}|}{T_1 + T_2}$$

$$\frac{S_{max}}{T_4} = D$$

$$\frac{S_{max}}{T_1} = P - D$$

$$Q = D \cdot T = P \cdot (T_1 + T_2)$$

Total Cost (over one cycle T)

① Ordering cost ($\$A/\text{order}$)

$$\$A * 1$$

② Inventory cost (i : interest rate, c : item cost, ic : holding cost) for one cycle

$$i \cdot c \times \triangle = i \cdot c \cdot \frac{1}{2} (I_{max} (T_2 + T_3))$$

③ Shortage cost

{ fixed shortage cost: $\$W/\text{unit}$
variable shortage cost $\$W_i/\text{unit} \cdot \text{time}$

$$\$W \cdot S_{max}$$

$$\$W_i \cdot (\triangle + \triangle) = W_i \cdot (\frac{1}{2} S_{max} (T_1 + T_4))$$

④ Item Cost: $\$C/\text{unit}$

$$\$C \cdot Q$$

Total Cost: ① + ② + ③ + ④
over one cycle

$$\Rightarrow \frac{① + ② + ③ + ④}{T}$$

$$\frac{I_{max} + S_{max}}{T_3 + T_4} = D \quad I_{max} = D(T_3 + T_4) - S_{max}$$

$$= D \left(+ \frac{S_{max}}{D} \right)$$

$$= (P - D) \frac{Q}{P} - S_{max}$$

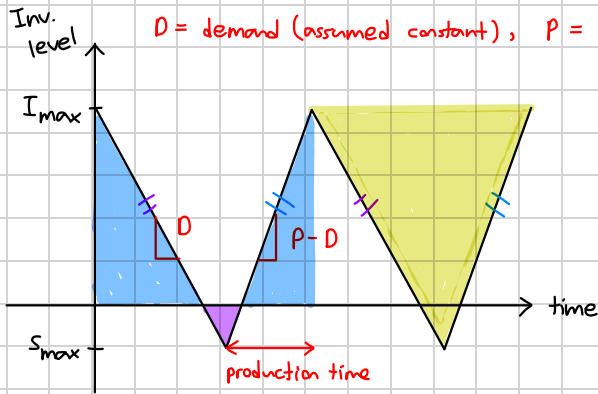
$$\left\{ \frac{2TC(Q, S_{max})}{2Q} = \phi \right.$$

$$\left. \frac{2TC(Q, S_{max})}{2S_{max}} = \phi \right.$$

$$TC(Q, S_{max}) = \frac{A \cdot D}{Q} + \frac{i \cdot c \cdot (Q(1 - D/P) - S_{max})^2}{2(Q)(1 - D/P)} + \frac{W \cdot S_{max} \cdot D}{Q} + \frac{W_i \cdot S_{max}^2}{2Q(1 - D/P)} + C \cdot D$$

Lecture 2/27/2024 : Inventory Model

Batch Production (Lot)



$P - D$: you'll only deal w/ inventory model if production rate > demand. $\therefore P > D$

$S_{\max}$: max shortage $I_{\max}$: max inventory

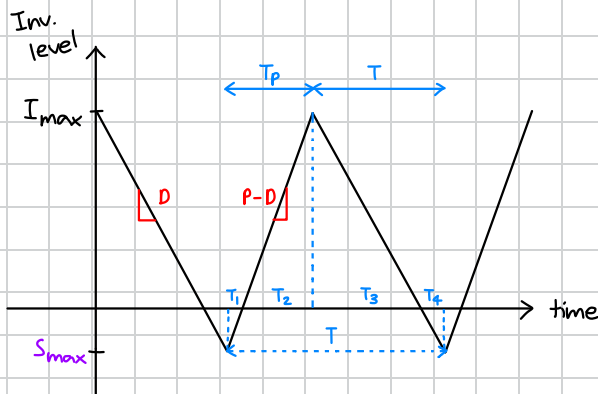
Q^* : EOQ (Economic Order Quantity)

→ AKA lot size / batch lot

→ size of triangle

→ can control $S_{\max}$ (how low)

total I. overtime total shortage overtime



T : cycle time

T_1 & T_4 : non-existing if S not allowed

T_1 : $S_{\max} \rightarrow 0$ shortage

T_2 : start accumulating inventory; $0 \rightarrow I_{\max}$

T_3 : stop production: $I_{\max} \rightarrow 0$

T_4 : start allowing back order

$$D = \frac{I_{\max} + S_{\max}}{T_3 + T_4}$$

$$P - D = \frac{I_{\max} + S_{\max}}{T_1 + T_2}$$

$$D = \frac{S_{\max}}{T_4}$$

$$P - D = \frac{S_{\max}}{T_1}$$

Variables: $Q, S_{\max}$

Constants: D, P where $(P > D)$

$$Q = \underset{\text{is demanded}}{D} \underset{\text{how much}}{T} = \underset{\text{is produced}}{P} \underset{\text{how much}}{T_p} = P(T_1 + T_2)$$

$$\text{Total Cost (Over one cycle } T) = ① + ② + ③ + ④$$

$$\text{Average Total Cost: } \frac{① + ② + ③ + ④}{T}$$

$Q, S_{\max}, P, D, A, i, c, W, W_1$

$$T = \frac{Q}{D}, I_{\max} = D(I_3 + I_4) - S_{\max}, S_{\max} = D \cdot T_4$$

$$I_{\max} = (P - D) \frac{Q}{P} - S_{\max}$$

① Ordering Cost (\$A/order)

② Inventory Cost (i : interest rate c : item cost)

$$\$A * 1 (\text{time period})$$

$$\underbrace{i \cdot c}_{\text{holding cost}} \cdot \triangle = i \cdot c \cdot \frac{1}{2} (I_{\max} (T_2 + T_3))$$

③ Shortage Cost

④ Item Cost $\$C/\text{unit}$

fixed shortage cost $\$W/\text{unit}$
variable shortage cost $\$W_1/\text{unit} \cdot \text{time}$

$$\$W \cdot S_{\max}$$

$$\$W_1 \cdot (\triangle + \nabla) = W_1 \cdot \left(\frac{1}{2} S_{\max} (T_1 + T_4) \right)$$

$$\begin{aligned} TC(Q, S_{\max}) = & \frac{A \cdot D}{Q} + \frac{i \cdot c \left(Q - \left(1 - \frac{D}{P}\right) - S_{\max} \right)^2}{2(Q) \left(1 - \frac{D}{P}\right)} \\ & + \frac{W \cdot S_{\max} \cdot D}{Q} + \frac{W_1 \cdot S_{\max}^2}{2Q \left(1 - \frac{D}{P}\right)} \\ & + C \cdot D \end{aligned}$$

$$TC(Q, S_{max}) = \frac{A \cdot D}{Q} + C \cdot D + \frac{i \cdot C \cdot (Q(1-D/p) - S_{max})^2}{2Q(1-D/p)} + \frac{W \cdot S_{max} \cdot D}{Q} + \frac{W_1 S_{max}^2}{2Q(1-D/p)}$$

ordering cost item cost Inv. Cost
 ↑ ↑ ↑
 fixed shortage cost Variable shortage cost

Example: $C: \$30/\text{unit}$ $D = 350/\text{year}$
 $i: 0.18 \text{ annual}$ $A: \$10/\text{order}$
 $W: \$0.3/\text{unit}$ $W_1 = \$5/\text{unit-year}$

"Instant Delivery"

all products are received in "one shipment"

$$\begin{cases} Q^* \\ S_{max}^* \end{cases}$$

P, D are constant $P > D$

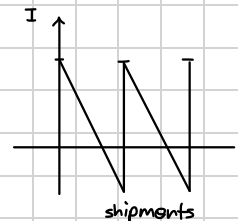
A, e, i, w, w_1 : cost coefficients

Instant Delivery $\Rightarrow p(\text{product. rate}) \rightarrow \infty$
 $D/p \Rightarrow 0$

$$TC(Q, S_{max}) = \frac{10 \times 350}{Q} + 30 \times 350 + \frac{0.18(30)(Q - S_{max})^2}{2Q} + \frac{0.3 \times S_{max}(350)}{Q} + \frac{5 S_{max}^2}{2Q}$$

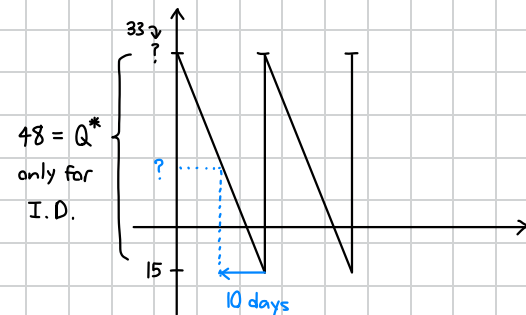
$$= 10500 + 2.7Q - 5.4 S_{max} + \frac{5.2 S_{max}^2}{Q} + \frac{105 S_{max}}{Q} + \frac{3500}{Q}$$

$$\frac{2TC(Q, S_{max})}{Q} = \phi \quad \frac{2TC(Q, S_{max})}{2S_{max}} = \phi$$



$$\begin{cases} \frac{2TC(Q, S_{max})}{2Q} = 2.7 - 5.2 \frac{S_{max}^2}{Q^2} - \frac{105 S_{max}}{Q^2} - \frac{3500}{Q^2} = 0 \\ \frac{2TC(Q, S_{max})}{2S_{max}} = -5.4 + \frac{5.2(2) S_{max}}{Q} + \frac{105}{Q} = 0 \end{cases}$$

$$Q^* = 47.83 \Rightarrow 48 \quad S_{max}^* = 14.82 \Rightarrow 15$$



Reordering point $\cdot$ LT (lead time) = 10 days

200 days/year*

$D = 350/\text{year}$

$D \cdot T = Q$

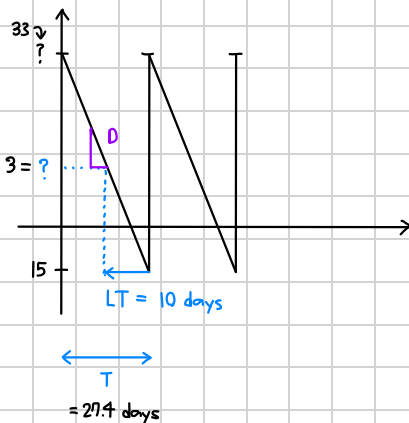
$350/\text{year} \cdot T = 48$

$T = 0.137 \text{ year} \times 200 \text{ days/year}$
 $= 27.4 \text{ days}$

$$\frac{x}{10 \text{ days}} = D = \frac{350/\text{year}}{200 \text{ day/year}} = 1.75/\text{day} = D$$

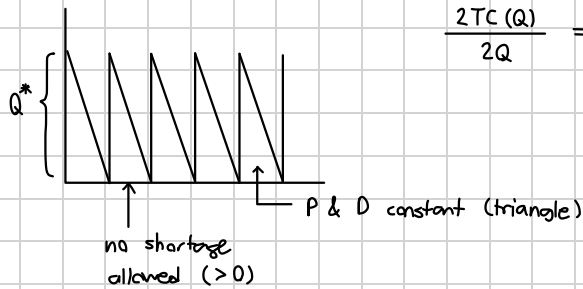
$$x = 1.75/\text{day} \times 10 \text{ days} = 17.5 \text{ units} \approx 18 \text{ units}$$

Re-ordering point = 3*



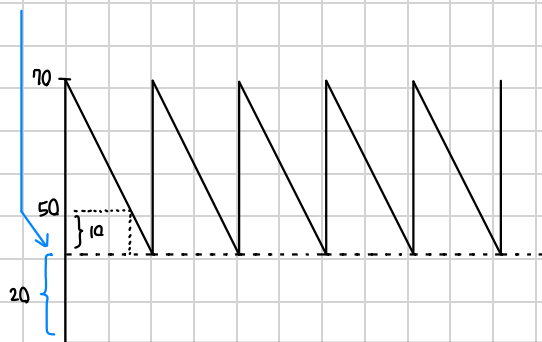
Classic EOQ { Instant Delivery $p \rightarrow \infty$
 No shortage allowed $S_{\max} = \phi$, $\frac{D}{P} \Rightarrow 0$

$$TC(Q) = \frac{AD}{Q} + C \cdot D + \frac{i \cdot CQ}{2}$$



$$\frac{2TC(Q)}{2Q} = \phi \quad Q^* = \sqrt{\frac{2AD}{i \cdot C}}$$

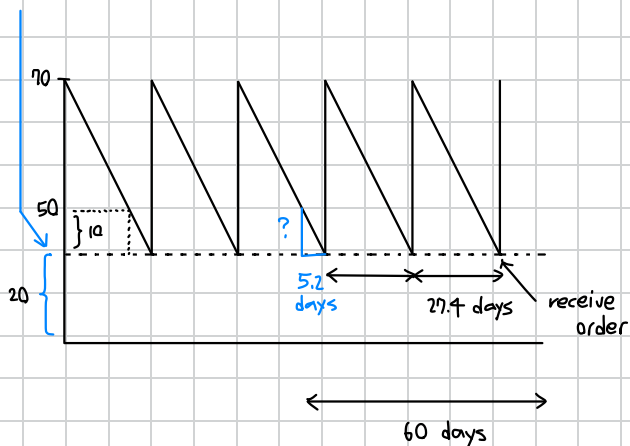
Safety Stock



say if reordering point,
 then new r.p. $\Rightarrow 10 + 20 = 30$

safety stock = 20.

$$Q^* = 50$$



$\Rightarrow$ Lead time is $>$ cycle time (T)

$$T = 27.4 \text{ days} \quad D = 350/\text{year} \Rightarrow \frac{350}{200} = 1.75 \text{ days}$$

Lead time = 60 days

reordering point

$$\Rightarrow 60 - 27.4 \times 2 = 5.2 \text{ days}$$

$$5.2 \text{ days} \times 1.75/\text{day} = 9.1 \text{ unit} \approx 9 \text{ unit}$$

re-ordering point = 6 in shortage.

ABC Analysis:

A group: 20% products $\Rightarrow$ 80% profit $\swarrow$ value of profit margin is high
 $\searrow$ high volume

Price Break Problem

Price $\Leftrightarrow$ Order Quantity Q^*

$0 < Q < 5000$	\$12/unit	$D = 250,000 / \text{yr}$
$5000 \leq Q < 20,000$	\$11/unit	$A = \$100/\text{order}$
$20,000 \leq Q < 40,000$	\$10/unit	$i = 0.24 \text{ annually}$
$40,000 \leq Q$	\$9/unit	Instant delivery
		No shortage allowed

Solution:

Instant Delivery $P \rightarrow \infty$
 No shortage allowed $S_{\max} = 0$ } classical EOQ

$$TC(Q) = \frac{AD}{Q} + CD + \frac{i \cdot C \cdot Q}{2}$$

$$Q^* = \sqrt{\frac{2AD}{i \cdot C}} = \sqrt{\frac{2 \times 100 \times 250,000}{0.24 \times 12}} = 4166$$

1). $C = \$12/\text{unit}$

$$TC(Q^*, \$12) = TC(4166, \$12) = \underline{\$3,012,000}$$

2). $C = \$11/\text{unit}$

$$TC(Q^*, \$11) = TC(5000, \$11) = \underline{\$2,761,000}$$

$$Q^* = \sqrt{\frac{2 \times 100 \times 250,000}{0.24 \times 11}} = 4352 < 5,000$$

set $Q^* = 5,000$

3). $C = \$10/\text{unit}$ $Q^* = 4564 < 20,000$
 set $Q^* = 20,000$

$$TC(20,000, \$10) = \underline{\$2,525,250}$$

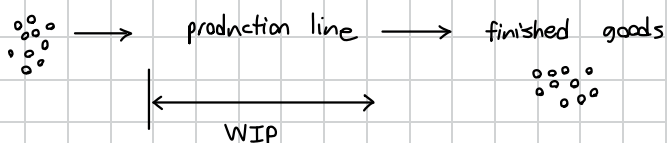
4). $C = \$9/\text{unit}$ $Q^* = 4811 < 40,000$
 set $Q^* = 40,000$

$$TC(40,000, \$9) = \underline{\$2,293,285} \longrightarrow TC(40,000, \$9) = \frac{\$100 \times 250,000}{40,000} + \$9 \times 250,000 + \frac{0.24(\$9) 40,000}{2}$$

Storage Capacity

$$T = \frac{40,000}{250,000} \text{ year}$$

- Inventory - Raw materials
 - Finished goods
 - WIP (work in progress)



0 wip = no production / empty (not possible)

We try to LEAN WIP to ↓ congestion

Lecture 3/7/2024 - Price Break Problem

→ Price depends on order quantity Q^*

Say.....

$0 < Q < 5,000$	\$12/unit
$5,000 \leq Q < 20,000$	\$11/unit
$20,000 \leq Q < 40,000$	\$10/unit
$40,000 \leq Q$	\$9/unit

$$\begin{aligned} D &= 250,000 \text{ / year} \\ A &= \$100 \text{ / order} \\ i &= 0.24 \text{ annually} \end{aligned}$$

Instant Delivery
No shortage allowed

Solution:

instant delivery $P \rightarrow \infty$
no shortage allowed $S_{\max} = 0$ } classical EOQ:

$$TC(Q) = \frac{AD}{Q} + CD + \frac{i \cdot C \cdot Q}{2}$$

for 1).
 $\frac{AD}{Q} \rightarrow \frac{100 \times 250,000}{4166} \rightarrow 6000$
 $CD \rightarrow 250,000 \times 12 \rightarrow 3,000,000$
 $\frac{i \cdot C \cdot Q}{2} \rightarrow \frac{0.24 \times 12 \times 250,000}{2} \rightarrow 3,600,000$
 $Q^* = \sqrt{\frac{2AD}{i \cdot C}} \rightarrow 4166$

1). $p = \$12/\text{unit}$

$$Q^* = \sqrt{\frac{2 \times 100 \times 250,000}{0.24 \times 12}} \approx 4166$$

$$TC(Q^*, \$12) = TC(4166, \$12) = \$3,012,000$$

3). $p = \$10/\text{unit}$

$$Q^* = \sqrt{\frac{2 \times 100 \times 250,000}{0.24 \times 10}} \approx 4564 < 20,000$$

set Q^* to 20,000

$$TC(Q^*, \$10) = TC(20,000, \$10) = \$2,525,250$$

2). $p = \$11/\text{unit}$

$$Q^* = \sqrt{\frac{2 \times 100 \times 250,000}{0.24 \times 11}} \approx 4352$$

set Q^* to 5,000

$$TC(Q^*, \$11) = TC(5000, \$11) = \$2,761,600$$

4). $p = \$9/\text{unit}$

$$Q^* = \sqrt{\frac{2 \times 100 \times 250,000}{0.24 \times 9}} \approx 4811 < 40,000$$

✓ set Q^* to 40,000

$$TC(Q^*, \$9) = TC(40,000, \$9) = \$2,293,285$$

$$\rightarrow \frac{\$100 \times 250,000}{40,000} + \$9 \times 250,000 + \frac{0.24(\$9) \cdot 40,000}{2}$$

$T = \frac{40,000}{250,000} \text{ year}$

Storage Capacity

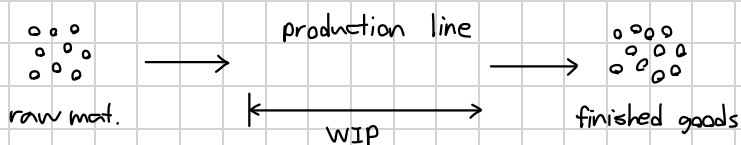
too much inventory → tighten up cashflow
(can't pay for operation costs)

Inventory

- Raw materials
- Finished goods
- WIP (work-in-progress)

JIT (ideal):

zero inventory (raw m.) $\Leftrightarrow$ directly into production line
(finished g.) $\Leftrightarrow$ directly shipped



zero WIP $\Leftrightarrow$ nothing in factory
cannot be 0, only reduced

Lecture 3/19/2024 : JIT

Just In Time: raw materials → WIP → finished goods
(Toyota)

- short lead time (req.)
- high quality products (req.) (everyone is Q.C. expert)
- steady demand (benefits)
- low cost (benefits)
- not implementable / fails if low quality / no constant demand / long lead time.

Ideal: no raw material / finished good inventories
(materials arrive & moved directly to the line: switch to WIP.) +
(finished goods get loaded & exported right away).

→ not a "solution" but a continuation of improvement (foundation).

Pull System

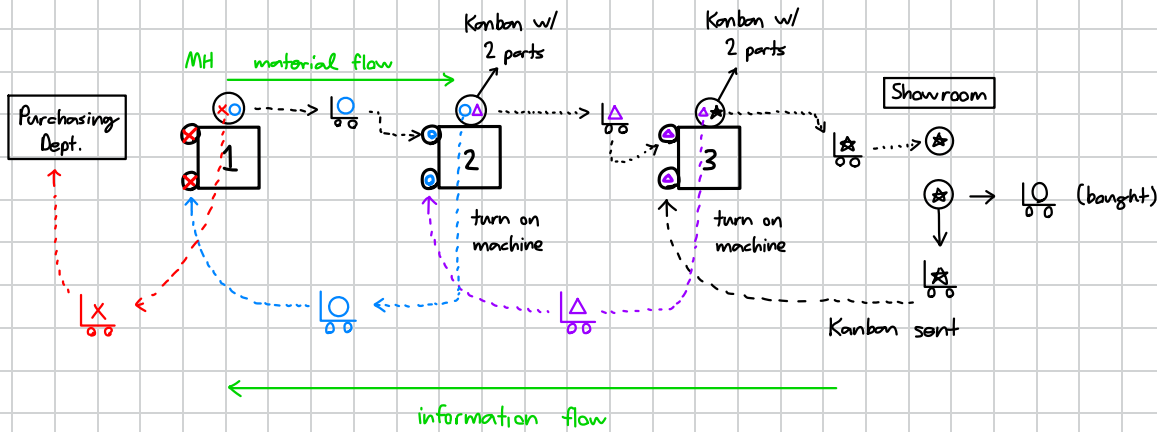
→ Driven by demand

Push System

→ Driven by forecasting

Kanban System: location, program code (for CNC), part ID, where to process

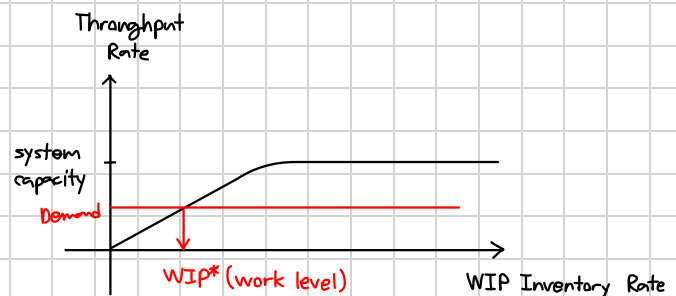
→ Push (driven by forecasting)



	push	pull
no input. (materials)	no production (acceptable)	shortage (express Kanban)
no demand	locate more space to store extra invent. line will eventually stop; Machine Blocked	no free Kanban (no drive / action)

express Kanban: sent to a previous station to prioritize a part; an alert to the chain.

machine blocked solution: buffer (individual sals.)
→ create traffic



Lecture 3/28/2024: Scheduling

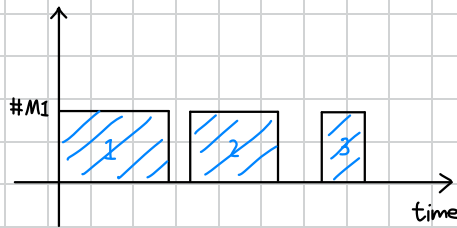
Single Machine

→ 1 →

★ no setup time (fixed) can be part of p_i

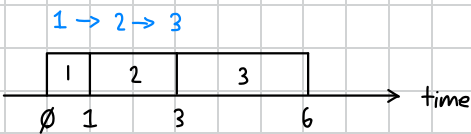
	i	1	2	3
Processing time:	p_i	1	2	3
Due date:	d_i	6	5	3
Arrival time:	a_i	$\emptyset$	$\emptyset$	$\emptyset$

Ghantt Chart: plot out time V.S. all resources



early vs. late
lateness $\leq 0, \geq 0$
tardiness ≥ 0 (on time or late)

Shortest Processing Time (SPT): shortest processing first



completion time = 6

Mean Flow Time: how much delay in the system; MFT = level of WIP & delay
→ ↑ MFT, ↑ wait time & longer queue

Trade off between MFT & Tardiness (no perfect solution).

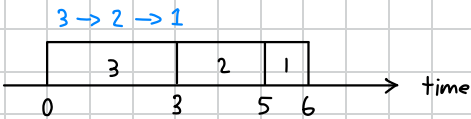
$$\text{max tardiness} = \max \{ \emptyset, \emptyset, (6-3) \} = 3$$

$$\text{mean flow time} = (1 + 3 + 6) / 3 = 10/3$$

(time in system)

Single Machine: all jobs arrive at time $\emptyset$, no setup time

Earliest Due Date (EDD): earliest due date first



completion time = 6

SPT → Minimize MFT (e.g. express lanes)
EDD → Minimize Max Tardiness (e.g. assignments)

$$\text{max tardiness} = \max \{ \emptyset, \emptyset, \emptyset \} = 0$$

$$\text{MFT} = (6 + 3 + 5) / 3 = 14/3$$

Parallel Machines

○ ○ → 2 → ○ ○

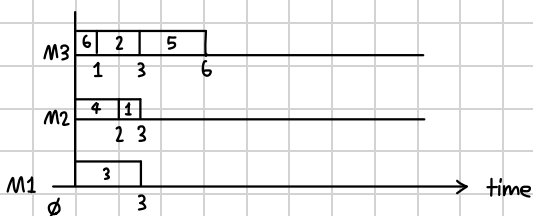
1

3

i	1	2	3	4	5	6
p_i	1	2	3	2	3	1
d_i	6	5	3	4	8	1
a_i	$\emptyset$	$\emptyset$	$\emptyset$	0	0	0

Jackson's Rule:

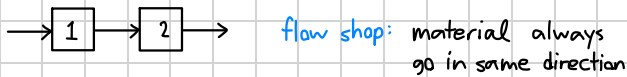
Broader category of EDD → minimize max tardiness : 6 → 3 → 4 → 2 → 1 → 5



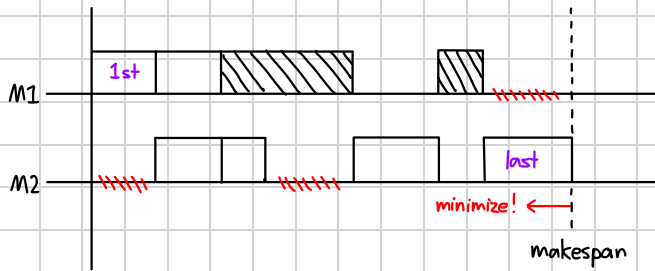
→ Task 4 could be assigned to machine 2 or 3.
→ Task 5 could be assigned to any
→ Max tardiness same regardless of ghantt chart configuration.

$$\text{max tardiness} = \max \{ \emptyset, \emptyset, \dots, \emptyset \} = \emptyset$$

Serial Machines



Minimize Makespan (completion time of all jobs through all processes),



Johnson's Algorithm

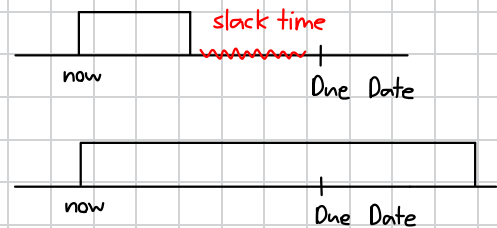
- Shortest first process $\Rightarrow$ early
- Shortest second process $\Rightarrow$ late

$$\text{Critical Ratio} = \frac{\text{time remaining}}{\text{work remaining}}$$

→ Prioritize task w/ low C.R.

Flow Shop 2 Machine Problem:

i	1	2	3	4	5	
P_{11}	2	3	4	4	6	→ 1 → 2 → * all jobs arrive at time $\emptyset$ * no setup time
P_{22}	5	1	2	8	7	



$$\text{Min} \{ P_{ij} \} = 1 = P_{22} ;$$

second process $\rightarrow$ late: 2 & P_{22} / P_{12} removed

$$\text{Min} \{ P_{ij} \} = 2 = P_{32} ;$$

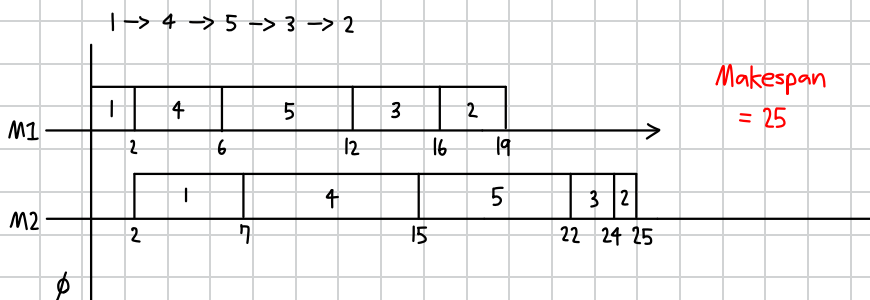
second process $\rightarrow$ late: 3 2 & P_{23} / P_{13} removed

$$\text{Min} \{ P_{ij} \} = 2 = P_{11} ;$$

first process $\rightarrow$ early: 1 3 2 & P_{11} / P_{12} removed

$$\text{Min} \{ P_{ij} \} = 4 = P_{41} ;$$

second process $\rightarrow$ early: 1 4 5 3 2 , 5 remaining & filled.



SPT, EDD, FIFO, LIFO

Lecture April 9th

CORE LAP - Computerized Relationship Layout Planning

inputs { Relationship Chart
Size of Dept (Blocks) } constructive procedure

Dept	1	2	3	4	5	Area	TCR
1	-	V	E	I	V	1	$U+E+I+U = 2+4+5+2$
2	V	-	0	0	V	2	$2+3+2+2 = 10$
3	E	0	-	A	0	2	17
4	I	0	A	-	V	1	15
5	V	V	0	V	-	3	9

$A=6$ $E=5$ $I=4$ $0=3$ $V=2$ $X = -100$

(1) Translation Phase

Total Closeness Rating (TCR) = $\sum$ Rels

(2) Selection Phase

Dept. ③ First Dept - Highest TCR

Tie Breaking Rules (1) largest area
(2) lowest ID #

② Fourth Dept. -

⑤ Fifth Dept.

"A" - 1st

"A" - 2nd

"A" - 3rd

"E" -

Dept. ④ Second Dept - "A Rel. w/ First Dept.

If tied, ① highest TCR

② largest area

③ lowest ID #

If none, "E" Rel. w/ First Dept

3 → 4 → 1 → 2 → 5

(3) Placement Phase

Placement Rating (PR)

= $\sum$ Rel (w/ common wall)

= Rel (3, 4) = A = 6

4
3 3

4	1
3	3

1	4
	3 3

4
3 3
1

$$\begin{aligned} PR &= R(1, 4) + R(1, 3) \\ &= I + E = 9 \end{aligned}$$

highest

$$\begin{aligned} PR &= R(1, 4) \\ &= I = 4 \end{aligned}$$

$$\begin{aligned} PR &= R(1, 3) \\ &= E = 5 \end{aligned}$$

2	2
4	1
3	3

2	4	1
2	3	3

4	1	2
3	3	2

4	1
3	3
2	2

$$\begin{aligned} PR &= (2, 4) + (2, 1) \\ &= 0 + 0 \\ &= 5 \end{aligned}$$

$$\begin{aligned} PR &= (2, 4) + (2, 3) \\ &= 0 + 0 \\ &= 6 \end{aligned}$$

highest

$$\begin{aligned} PR &= (2, 1) + (2, 3) \\ &= 0 + 0 \\ &= 5 \end{aligned}$$

$$\begin{aligned} PR &= (2, 3) \\ &= 0 \\ &= 3 \end{aligned}$$

5	5	5
2	4	1
2	3	3

5	2	4	1
5	2	3	3
5			

$$\begin{aligned} PR &= (5, 1) + (5, 4) \\ &+ (5, 2) = u + u + u \\ &= 6 \end{aligned}$$

highest

$$\begin{aligned} PR &= (5, 2) \\ &= u = 2 \end{aligned}$$

2	4	1
2	3	3
5	5	5

$$PR = (5, 1) + (5, 3) = u + 0 = 5$$

(4) Scoring Phase

5	5	5
2	4	1
2	3	3

$\sum Rel. \times Dist.$

↑
closest walls
rectilinear

12 13 14 15 23 24 25
 $1 \times 0 + \emptyset + \emptyset + \emptyset + \emptyset + \emptyset + \emptyset$

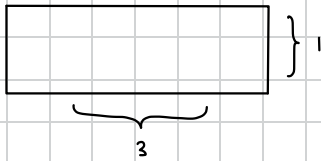
34 35 45
 $\emptyset + 1 \times 0 + \emptyset$



$1 + 0 = 2 + 3 = 5$
 the lower the better,
 (minimum problem)

1	2
3	4

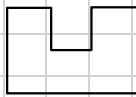
$dist(1, 4) = 1$



keep dept. size
 ratios < 1/3



acceptable



questionable



problematic

receiving +
 shipping center
 TCR ideal ✓
 but IRL ✗

Lecture 4th: Sequencing & Scheduling

Givens: a collection of jobs remaining to be processed on a collection of machines

Problem: how to sequence the given jobs to optimize some specified criterion

Common objective: maximize the throughput rate of the job shop

Flow time of job i = amount of time that job i spends in the production system

⇒ **mean flow time:** the arithmetic mean of flow times for all n jobs.

Tardiness is the positive difference btwn completion time & due date of a job.

⇒ job's tardiness is zero if completed before due date

⇒ a **tardy job** is completed after its due date

⇒ **common scheduling objectives:** max. tardiness, mean tardiness, & no. of tardy jobs.

Lateness is the diff. btwn the job completion time & its due date

⇒ can be either (+) or (-): job completed before due date

Makespan is the time to complete all n jobs

First Come First Served (FCFS): jobs are processed in the sequence they entered the queue

Shortest Processing Time (SPT): jobs are sequenced in increasing order of their processing times;

⇒ job w/ shortest P.T. first, the next shortest P.T. as second, and so on.

Earliest Due Date (EDD): jobs are sequenced in increasing order of their due dates;

⇒ job w/ shortest D.D. first, the next shortest D.D. as second, and so on.

Critical Ratio (CR):
$$\frac{\text{remaining time until due date (due date - current time)}}{\text{processing time of the job (at a machine)}}$$
, schedule job w/ smallest C.R.

★ Single Machine Problem ★

→ P.T. & D.D. of each job are given constants

→ All jobs arrive simultaneously

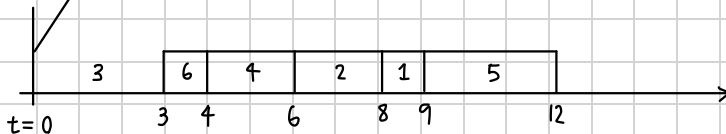
→ Setup time is fixed

→ **SPT minimizes mean flow time**

→ **EDD minimizes maximum tardiness**

Job #	1	2	3	4	5	6
Processing time	1	2	3	2	3	1
Due time	6	5	3	4	8	1
Arrival time	0	0	0	0	0	0

Gantt Chart



$t=6$). Job 1, 2, 5: $CR_1 = \frac{6-6}{1} = 0$, $CR_2 = \frac{5-6}{2} = -0.5$, $CR_5 = \frac{8-6}{3} = 0.67$

$t=8$). Job 1 & 5: $CR_1 = \frac{6-8}{1} = -2$, $CR_5 = \frac{8-8}{3} = 0$, then CR_5 last.

$t=0$). Job 1~6; no tie-breaking rules: job 3 or 6 first

$$CR_1 = \frac{6-0}{1} = 6 \quad CR_3 = \frac{3-0}{3} = 1 \quad CR_5 = \frac{8-0}{3} \approx 2.67$$

$$CR_2 = \frac{5-0}{2} = 2.5 \quad CR_4 = \frac{4-0}{2} = 2 \quad CR_6 = \frac{1-0}{1} = 1$$

$t=3$). Job 1~2, 4~6:

$$CR_1 = \frac{6-3}{1} = 3 \quad CR_4 = \frac{4-3}{2} = 0.5 \quad CR_6 = \frac{1-3}{1} = -2$$

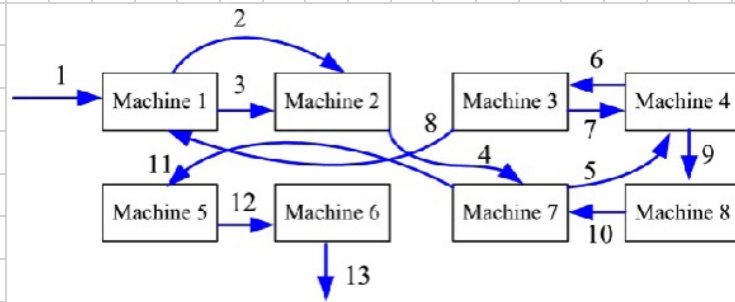
$$CR_2 = \frac{5-3}{2} = 1 \quad CR_5 = \frac{8-3}{3} \approx 1.67$$

$t=4$). Job 1~2, 4~5: $CR_1 = \frac{6-4}{1} = 2$

$$CR_2 = \frac{5-4}{2} = 0.5 \quad CR_4 = \frac{4-4}{2} = 0 \quad CR_5 = \frac{8-4}{3} \approx 1.33$$

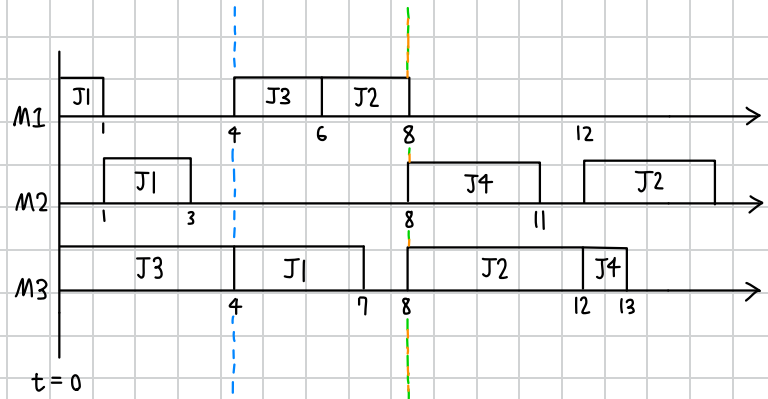
Flow shop: each of the n jobs must be processed through the m machines in the same order, & each job is processed exactly once on each machine (assembly line).

Job shop: not all jobs are assumed to require exactly m operations, & some jobs may require multiple operations on a single machine. Furthermore, each job may have a diff. required sequencing of operations. Can be extremely complex & thus has all-purpose solution algorithm.



★ Job Shop Scheduling Problem with Dynamic Arrival: ★ → Apply EDD & CR

✓ Job 1: 10	Machine 1 → Machine 2 → Machine 3 (1 hour) (2 hours) (3 hours)	Arrival time = 0 Due time =
✓ Job 2: 10	Machine 1 → Machine 3 → Machine 2 (2 hours) (4 hours) (3 hours)	Arrival time = 4 Due time = 15
✓ Job 3: 10	Machine 3 → Machine 1 (4 hour) (2 hours)	Arrival time = 0 Due time = 8
Job 4: 10	Machine 2 → Machine 3 (3 hour) (1 hour)	Arrival time = 8 Due time = 18



$t = 4$. J3 → M1 8 ✓
J2 → M1 15
J1 → M3

$t = 8$. J4 → M2
J2 → M3
 $t = 12$. J4 → M3
J2 → M2

Layout Design · (I) Types of Layouts

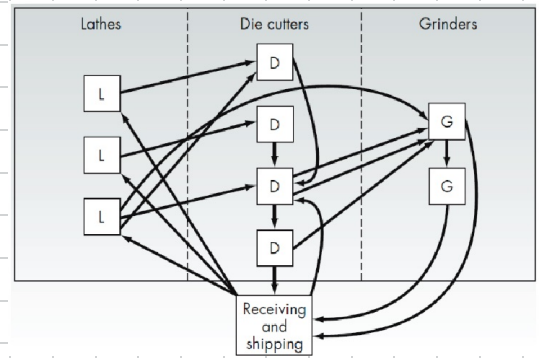
Fixed Position Layout. for products that are too bulky & heavy to move (e.g. planes, ships, & rockets). The product remains fixed & the layout is determined later based on product size & shape.

Product Layout (Or Product Flow Layout):

- machines are organized to conform the sequence of operations required to produce the product
- typical layout for high-volume standardized production
- an assembly line (or transfer line)
- desirable for flow-type mass production & provide the fastest cycle times in such environ.

Process Layout (Or Job Shop Layout)

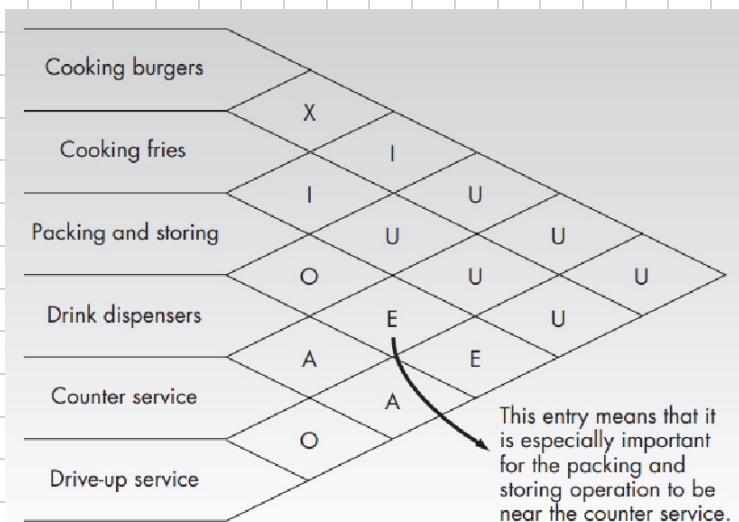
- Groups of machine having similar functions
- Common for small/medium volume manufacturers
- Most effective when there is a wide variation in the product mix.
- Each product has a diff. routing sequence associated with it.
- Flow patterns are highly variable.



Group Technology Layout (or Cellular Layout)

- Parts must be identified & grouped in part families based on similarities in manufacturing function or design.
 - Each family requires similar processing
 - Each cell of machines corresponds to a particular part family
 - Suitable for large firms producing a wide variety of parts in moderate to high volumes.
- process - layout product - layout

Layout Design - (2) Charts



Letter codes for closeness ratings:

- A: Absolutely necessary
- E: Especially important
- I: Important
- O: Ordinary importance
- U: Unimportant
- X: Undesirable

FIGURE 11-3

From-to chart showing distances between six department centers (measured in feet)

FIGURE 11-4

From-to chart showing number of materials handling trips per day

To From	Saws	Milling	Punch press	Drills	Lathes	Sanders
Saws		18	40	30	65	24
Milling	18		38	75	16	30
Punch press	40	38		22	38	12
Drills	30	75	22		50	46
Lathes	65	16	38	50		60
Sanders	24	30	12	46	60	

To From	Saws	Milling	Punch press	Drills	Lathes	Sanders
Saws		43	26	14	40	
Milling			75	60		23
Punch press					45	16
Drills		22			28	
Lathes		45		30		60
Sanders		12				

Layout Design - (3) Computerized Layout Techniques

Computerized Relative Allocation of Facilities Technique (CRAFT)

→ Improvement routine (requires the user to specify an initial layout)

Objective: minimize total transportation cost of a layout

Transportation cost = product of
(the cost to move a unit load from a department to another)
&
(the [rectilinear] distance btwn. the 2 departments' centroid)

→ Based on an initial layout, CRAFT considers exchanging the positions of adjacent departments & computes the materials handling cost of the resulting exchange; the pairwise interchange that results in the greatest cost reduction is chosen by the program.

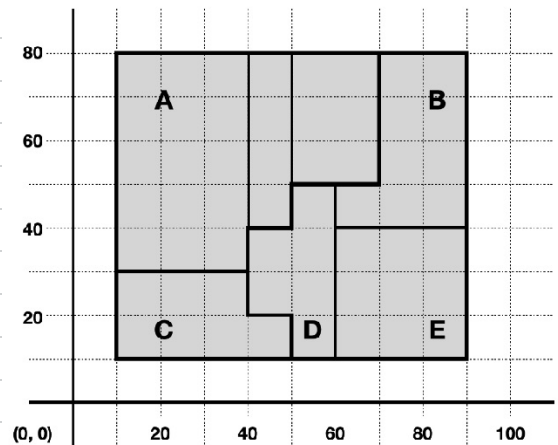
Let R be any region in the plane and $A(R)$ is its area. Also, suppose that R can be subdivided into k rectangles.

The centroid of R is given by $\bar{x} = \frac{M_x}{A(R)}$ & $\bar{y} = \frac{M_y}{A(R)}$

where $M_x = \sum_{i=1}^k \frac{x_{2i}^2 - x_{1i}^2}{2} (y_{2i} - y_{1i})$

$$M_y = \sum_{i=1}^k \frac{y_{2i}^2 - y_{1i}^2}{2} (x_{2i} - x_{1i})$$

Centroid
= coordinates of
the center of
gravity



< Department A > / Example

$$M_x = \frac{(40^2 - 10^2)}{2} (80 - 30) + \frac{(50^2 - 40^2)}{2} (80 - 40) + \frac{(70^2 - 50^2)}{2} (80 - 50) \approx 91,500$$

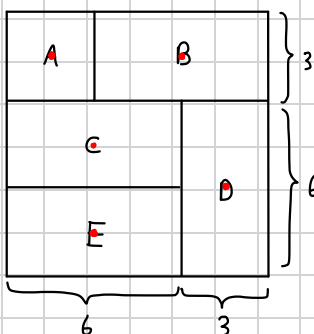
$$M_y = \frac{(80^2 - 30^2)}{2} (40 - 10) + \frac{(80^2 - 30^2)}{2} (50 - 40) + \frac{(80^2 - 70^2)}{2} (70 - 50) \approx 145,500$$

$$\bar{x} = \frac{91,500}{2500} = 36.6 \quad \bar{y} = \frac{145,500}{2500} = 58.2, \quad \text{centroid} = (36.6, 58.2)$$

Example 2:

Flow Chart:

use CRAFT to determine the score



	A	B	C	D	E
A			24		
B				17	18
C					5
D	20	12			10
E					

	A	B	C	D	E
A			45		
B				6	9
C					3
D	10.5	6			6
E					

score:

$$24 \times 4.5 + 17 \times 6 + 18 \times 9 + 5 \times 3 + 20 \times 10.5 + 12 \times 6 + 10 \times 6 = 729$$

if new layout beats (>) 729, then new is better!

Computerized Relationship Layout Planning (CORELAP) (Placement Phase)

→ Greedily places departments in the determined order while maximizing the placement rating / constructive procedure

→ Relationship chart, size of department (Blocks)

	1	2	3	4	5	Area	TCR
1	-	u	E	I	u	1	$u + E + I + u = 13$
2	u	-	0	0	u	2	10
3	E	0	-	A	0	2	17
4	I	0	A	-	u	1	15
5	u	u	0	u	-	3	9

$A=6, E=5, I=4, 0=3, u=2, x=-100$

Dept. 1

→ 3rd Dept. is one w/ "A" relation w/ 1st Dept.

- T.B.R.
- If none "A", then "A" w/ 2nd Depart.
- T.B.R.
- If none, "E" w/ 1st Dept.
- T.B.R.
- If none, "E" w/ 2nd Dept.
- T.B.R.
- "I", "0", "u"

1) Translation Phase:

→ find the Total Closeness Ratings (TCR)

2) Selection Phase:

→ 1st Dept. is one w/ highest TCR

Dept. 3 • Tie Breaking Rules (T.B.R.)

- (1). largest area
- (2). lowest area

Dept. 4

→ 2nd Dept. is one w/ "A" relation w/ 1st Dept.

- If tied: (1). highest TCR
- (2). largest area
- (3). lowest area
- If no "A". "E" w/ relation w/ 1st Dept.
- If tied: TCR + Tie Breaking R.

→ 4th Dept. is Dept. 2 & 5th Dept. is Dept. 5, since $TCR(2) > TCR(5)$.

→ Common wall length doesn't affect no. of relationships

3) Placement Phase:

Placement Rating (PR) / choose the highest

$$= \sum \text{Rel (w/ common wall)}$$

$$= \text{Rel}(3, 4) = A = 6$$

2	2
4	1
3	3

$$PR = (2, 4) + (2, 1)$$

$$= 0 + u$$

$$= 5$$

2	4	1
2	3	3

$$PR = (2, 4) + (2, 3)$$

$$= 0 + 0$$

$$= 6 \quad \checkmark \text{ highest}$$

4	1	2
3	3	2

$$PR = (2, 1) + (2, 3)$$

$$= u + 0$$

$$= 5$$

4	1
3	3
2	2

$$PR = (2, 3)$$

$$= 0$$

4	1
3	3

$$PR = R(1, 4) + R(1, 3)$$

$$= I + E = 9 \quad \checkmark \text{ highest}$$

1	4
	3
	3

$$PR = R(1, 4)$$

$$= I = 4$$

4
3
3
1

$$PR = R(1, 3)$$

$$= E = 5$$

4	1
3	3
2	2

$$PR = (2, 3)$$

$$= 0$$

$$= 3$$

5	5	5
2	4	1
2	3	3

$$PR = (5, 1) + (5, 4)$$

$$+ (5, 2) = u + u + u$$

$$= 6 \quad \checkmark \text{ highest}$$

5	2	4	1
5	2	3	3
5			

$$PR = (5, 2)$$

$$= u = 2$$

2	4	1
2	3	3
5	5	5

$$PR = (5, 1) + (5, 3)$$

$$= u + 0 = 5$$

4). Scoring Phase

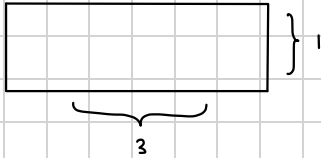
- $\sum Rel \times dist.$ (closest walls NOT centroid [CRAFT])
- lowest layout score is better!
- minimum problem
- neighboring (w/ common wall), then dist/score = 0

5	5	5
2	4	1
2	3	3

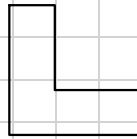
$$\begin{array}{cccccccccccc}
 1-2 & 1-3 & 1-4 & 1-5 & 2-3 & 2-4 & 2-5 & 3-4 & 3-5 & 4-5 \\
 1 \times u & + & \emptyset & + & \emptyset & + & \emptyset & + & \emptyset & + & 1 \times 0 & + & \emptyset \\
 \Rightarrow u + 0 & = & 5 & \text{(Layout score)}
 \end{array}$$

★ common practice:

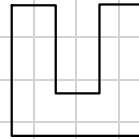
keep dept. size ratios $< \frac{1}{3}$



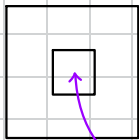
L-shape (Acceptable)



U-shape (Questionable)



highest TCR $\neq$ best



shipping/export center

how to exit?
helicopter?

How to calculate the RPW of a work element?:

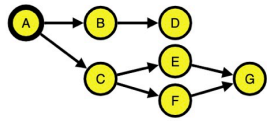
RPW of work element X := the sum of the operating time required for the element X and the times for all elements that must succeed that element.

high RPW : higher position
greater weight
/value

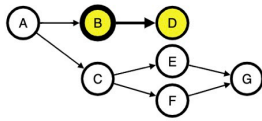
Example

$t_A = 3$ $t_B = 2$ $t_C = 4$ $t_D = 1$ $t_E = 2$ $t_F = 1$ $t_G = 5$

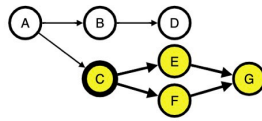
RPW_A = 18



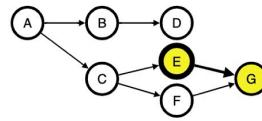
RPW_B = 3



RPW_C = 12



RPW_E = 7



$$3 + 2 + 4 + 1 + 2 + 1 + 5 = 18$$

$$t_B + t_D = 3$$

$$t_C + t_E + t_F + t_G = 12$$

$$t_E + t_G = 7$$

Exercise: Line Balancing

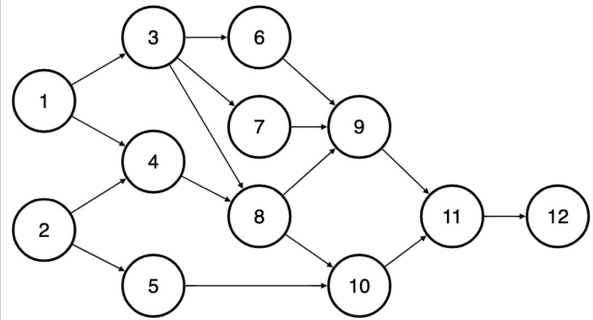
The precedence relationship and task times (in minutes) for assembling products are shown below. Set up an assembly line to assemble 60 units of product in an hour. Balance the line and calculate the delay.

Desired cycle time

$$C_d = \frac{1 \text{ hour}}{60 \text{ units}} = 1 \text{ unit / hr.}$$

Work Element	Precedence	Time (min)	RPW
1	-	0.2	$4 - 0.4 - 0.3 = 3.3$
2	-	0.4	$4 - 0.2 - 0.7 - 0.32 = 2.67$
3	1	0.7	$4 - 0.2 - 0.1 - 0.4 - 0.3 = 3.0$
4	1,2	0.1	1.97
5	2	0.3	1.3
6	3	0.11	1.0
7	3	0.32	1.21
8	3,4	0.6	1.87
9	6,7,8	0.27	0.89
10	5,8	0.38	$0.38 + 0.12 + 0.5 = 0.9$
11	9,10	0.5	$0.12 + 0.5 = 0.62$
12	11	0.12	0.12

total = 4 mins



Precedence Diagram

Decreasing order of RPW

Work Element	RPW	Time (min)	Precedence
1	3.30	0.2	-
3	3.00	0.7	1
2	2.67	0.4	-
4	1.97	0.1	1,2
8	1.87	0.6	3,4
5	1.30	0.3	2
7	1.21	0.32	3
6	1.00	0.11	3
10	1.00	0.38	5,8
9	0.89	0.27	6,7,8
11	0.62	0.50	9,10
12	0.12	0.12	11

Desired cycle time = 1min/unit

WS1 (1) Assign work element 1 (0.2 min) to WS1
(2) Assign work element 3 (0.7 min) to WS1

WS2 (3) Assign work element 2 (0.4 min) to WS2
(4) Assign work element 4 (0.1 min) to WS2
(6) Assign work element 5 (0.3 min) to WS2

WS3 (5) Assign work element 8 (0.6 min) to WS3
(7) Assign work element 7 (0.32 min) to WS3

(5-1) Assign work element 8 (0.6 min) to WS2
— Exceeds desired cycle time (X)
(5-2) Assign work element 8 (0.6 min) to WS3
— Precedence (3 and 4) satisfied. (O)

(7-1) Assign work element 7 (0.32 min) to WS1 — (X)
(7-2) Assign work element 7 (0.32 min) to WS2 — (X)
(7-3) Assign work element 7 (0.32 min) to WS3 — (O)

(3-1) Assign work element 2 (0.4 min) to WS1

— Exceeds desired cycle time (X)

(3-2) Assign work element 2 (0.4 min) to WS2 (O)

(4-1) Assign work element 4 (0.1 min) to WS1

— Precedence (1 and 2) not satisfied. (X)

(4-2) Assign work element 4 (0.1 min) to WS2
— Precedence (1 and 2) satisfied.

(6-1) Assign work element 5 (0.3 min) to WS1 — (X)

(6-2) Assign work element 5 (0.3 min) to WS2 — (O)

Desired cycle time = 1min/unit

WS1	WS2	WS3	WS4	WS5
1,3	2,4,5,6	8,7	10,9	11,12
$0.2+0.7 = 0.9$	$0.4+0.1 + 0.3+0.11 = 0.91$	$0.6+0.32 = 0.92$	$0.38+0.27 = 0.65$	$0.5+0.12 = 0.62$

>> True Cycle Time = max(0.9, 0.91, 0.92, 0.65, 0.62) = 0.92 min

>> Efficiency = $4/(5 * 0.92)$ = approx. 87%

>> Balanced delay = approx. 13%

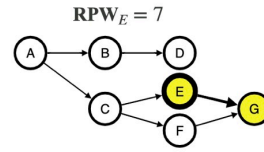
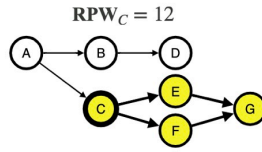
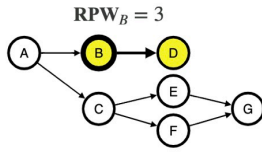
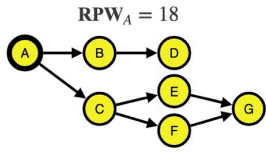
How to calculate the RPW of a work element?:

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high RPW : higher position
greater weight
/value

Example

$t_A = 3$ $t_B = 2$ $t_C = 4$ $t_D = 1$ $t_E = 2$ $t_F = 1$ $t_G = 5$



$$3 + 2 + 4 + 1 + 2 + 1 + 5 = 18$$

$$t_B + t_D = 3$$

$$t_C + t_E + t_F + t_G = 12$$

$$t_E + t_G = 7$$

Exercise: Line Balancing

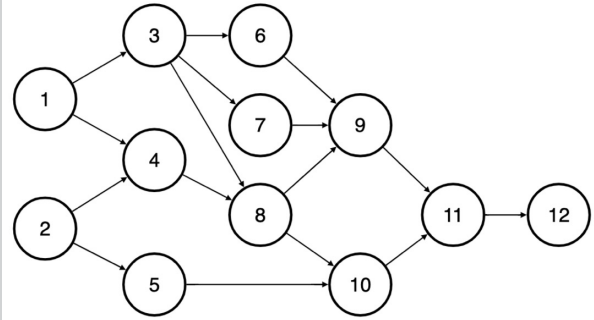
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6	3	0.11	1.0
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8	3,4	0.6	1.87
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10	5,8	0.38	$0.38 + 0.12 + 0.5 = 0.9$
11	9,10	0.5	$0.12 + 0.5 = 0.62$
12	11	0.12	0.12

total = 4 mins



Precedence Diagram

Decreasing order of RPW

Work Element	RPW	Time (min)	Precedence
1	3.30	0.2	-
3	3.00	0.7	1
2	2.67	0.4	-
4	1.97	0.1	1,2
8	1.87	0.6	3,4
5	1.30	0.3	2
7	1.21	0.32	3
6	1.00	0.11	3
10	1.00	0.38	5,8
9	0.89	0.27	6,7,8
11	0.62	0.50	9,10
12	0.12	0.12	11

Desired cycle time = 1min/unit

WS1

- (1) Assign work element 1 (0.2 min) to WS1
- (2) Assign work element 3 (0.7 min) to WS1

WS2

- (3) Assign work element 2 (0.4 min) to WS2
- (4) Assign work element 4 (0.1 min) to WS2
- (6) Assign work element 5 (0.3 min) to WS2

WS3

- (5) Assign work element 8 (0.6 min) to WS3
- (7) Assign work element 7 (0.32 min) to WS3

- (5-1) Assign work element 8 (0.6 min) to WS2 — Exceeds desired cycle time (X)
- (5-2) Assign work element 8 (0.6 min) to WS3 — Precedence (3 and 4) satisfied. (O)

- (7-1) Assign work element 7 (0.32 min) to WS1 — (X)
- (7-2) Assign work element 7 (0.32 min) to WS2 — (X)
- (7-3) Assign work element 7 (0.32 min) to WS3 — (O)

(3-1) Assign work element 2 (0.4 min) to WS1

— Exceeds desired cycle time (X)

(3-2) Assign work element 2 (0.4 min) to WS2 (O)

(4-1) Assign work element 4 (0.1 min) to WS1

— Precedence (1 and 2) not satisfied. (X)

(4-2) Assign work element 4 (0.1 min) to WS2 — Precedence (1 and 2) satisfied.

(6-1) Assign work element 5 (0.3 min) to WS1 — (X)

(6-2) Assign work element 5 (0.3 min) to WS2 — (O)

Desired cycle time = 1min/unit

WS1	WS2	WS3	WS4	WS5
1,3	2,4,5,6	8,7	10,9	11,12
$0.2+0.7 = 0.9$	$0.4+0.1 + 0.3+0.11 = 0.91$	$0.6+0.32 = 0.92$	$0.38+0.27 = 0.65$	$0.5+0.12 = 0.62$

>> True Cycle Time = max(0.9, 0.91, 0.92, 0.65, 0.62) = 0.92 min

>> Efficiency = $4/(5 * 0.92)$ = approx. 87%

>> Balanced delay = approx. 13%